Mathematics of the Rubik's Cube

By

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Regular Polyhedrons

the Platonic solids

The Platonic solids have faces made from regular polygons.

Platonic solids	Name	Faces	Edges	Vertices
	tetrahedron	4	6	4
	octahedron	8	12	6
	icosahedron	20	30	12
	cube	6	12	8
	dodecahedron	12	30	20

N=1

2x2x2

You can assign a position for one of the pieces to establish the orientation of the puzzle. There are 7! possible permutations of the remaining pieces. Each of those can be twirled in place to one of 3 orientations. There is an invariant on those twirls that means that once that of the original fixed one and 6 others are known, then that of the last is determined. Thus there are 3^6 possibilities for the twirls on the 7 pieces we are allowing to move. $N=7!\cdot 3^6=3674160$.

3x3x3

There are:

6 centers that don’t move

8 corners with 3 orientations

12 edges with 2 orientations

Pos = 3^8 x 8! X 2^{12} x 12!

With 7 corners twisted, the 8th is determined so /3

With 11 edges fixed, the 12th is determined so /2

Cannot swap an odd # of times so /2

Pos = (3^8 x 8! X 2^{12} x 12!)/(3x2x2) = 43 252 003 274 489 856 000

Or 43 quintillion

God’s Number

Assumptions:
God exists
God has a 3x3x3 Cube
There exists a number, G, such that for all 43 quintillion starting positions of the cube, the cube can always be solved in a number of moves less than or equal to G

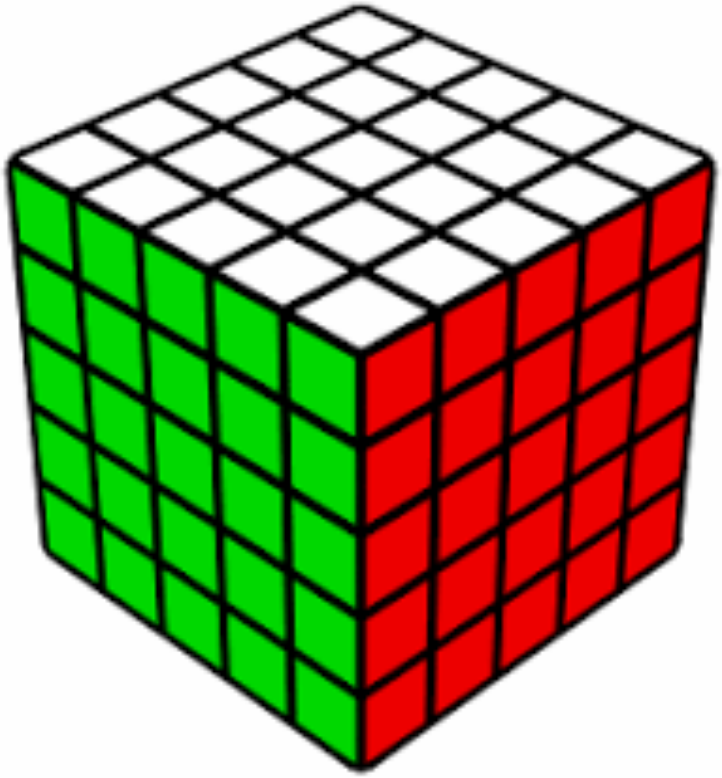
In 2015, G was found to equal 20

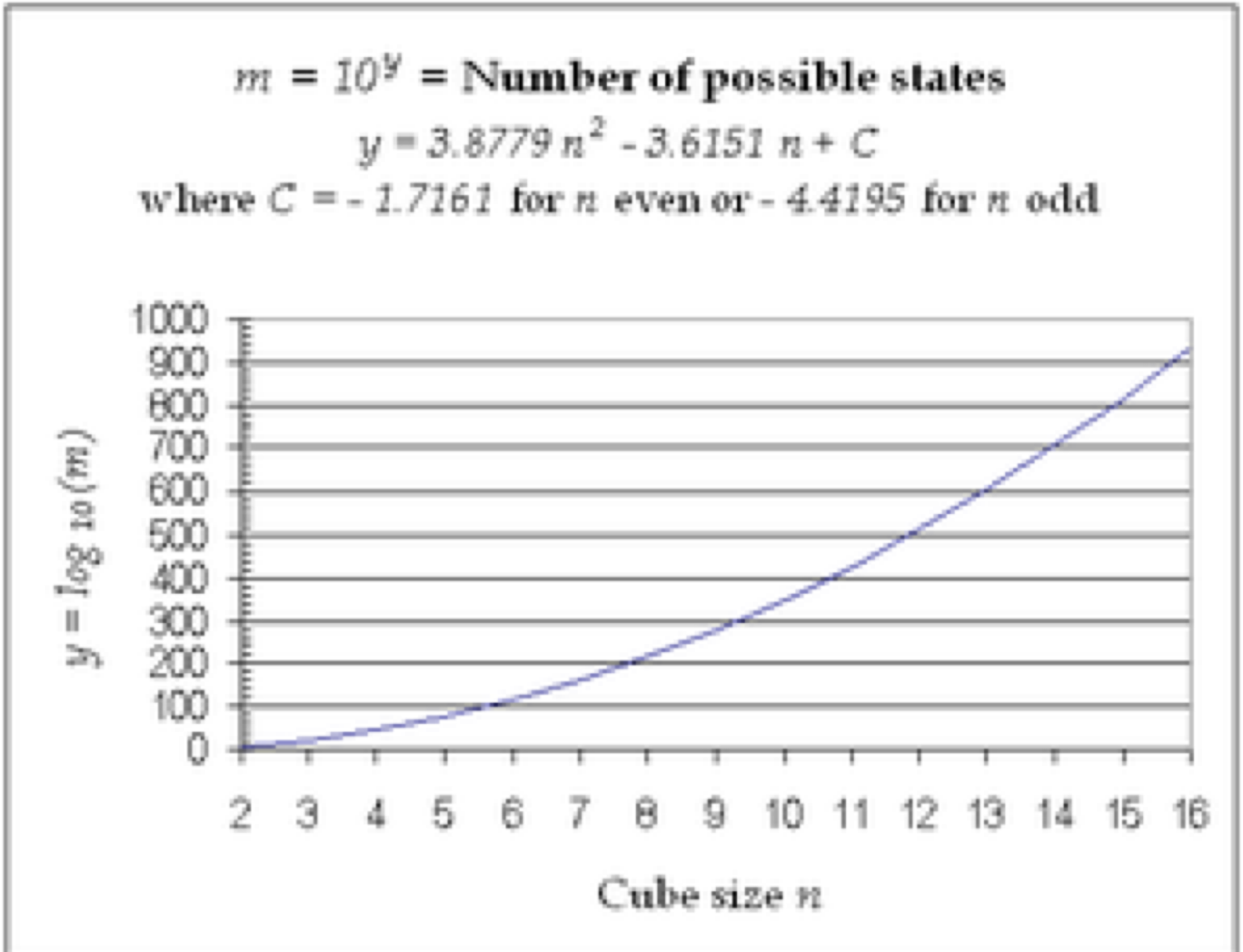
Devil’s Algorithm

Assumptions:
The Devil exists
The Devil has a 3x3x3 cube
There exists one algorithm so powerful that for all of the 43 quintillion starting positions of the cube, repeatedly applying this algorithm will always solve the cube.

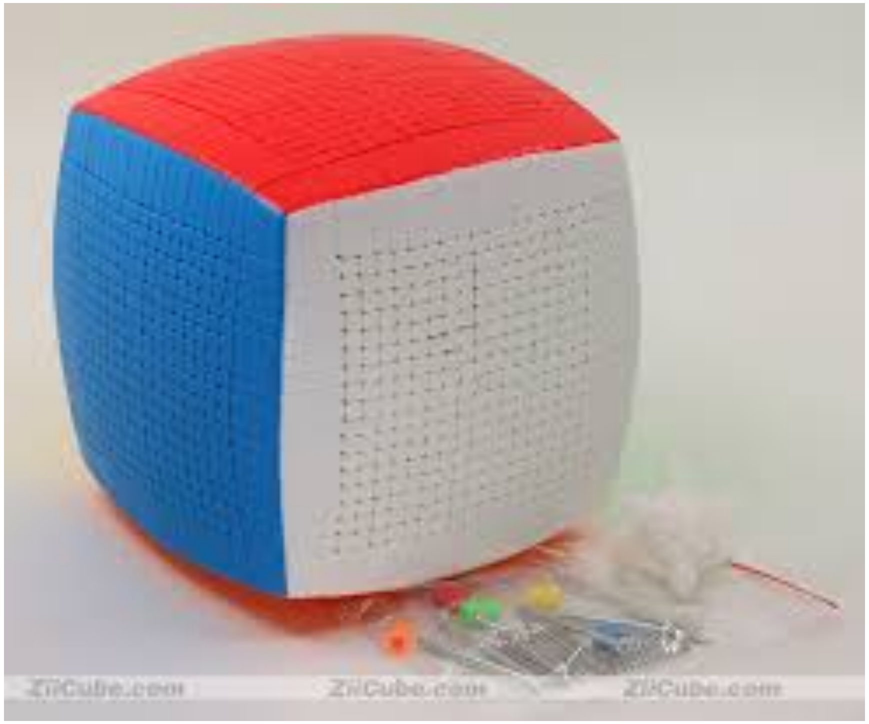
4x4x4 $N=7.4X10^{45}$

5x5x5 $N=2.8 \times 10^{72}$




m = 10^y = Number of possible states
y = 3.8779 n^2 - 3.6151 n + C
where C = - 1.7161 for n even or - 4.4195 for n odd
1000
900
800
700
600
500
400
300
200
100
0
y = log 10 (m)
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
Cube size n

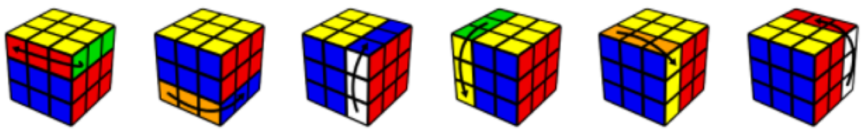
19x19x19 N=$1x10^{1327}$



Face Turns

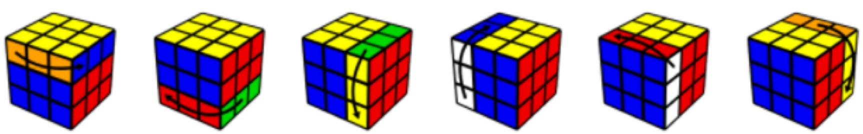
The basic moves are **U**p, **D**own, **R**ight, **L**eft, **F**ront, **B**ack.

Each move means to turn that side clockwise, as if you were facing that side.



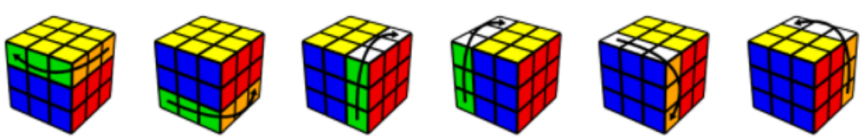
U **D** **R** **L** **F** **B**

An apostrophe (pronounced as *prime*) means to turn the face in the opposite direction (counterclockwise).



U' **D'** **R'** **L'** **F'** **B'**

The number 2 means to turn that face twice.



U2 **D2** **R2** **L2** **F2** **B2**

Note: Double moves such as U2 can be done clockwise or counterclockwise. The direction of the move can be specified by using U2 and U2' (to show finger tricks), but this is not always used.

Solution Methods CFOP

C cross – solve edge pieces on bottom layer

F first two layers – simultaneously solve the corner pieces of the bottom layer and the edge pieces on the middle layer

O orient last layer – get colors correct on last layer

P permute last layer – place all pieces correctly on last layer

Beginners Method

Cross – solve edge pieces on bottom layer
Solve corner pieces on bottom layer
Solve edge pieces on middle layer
Orient edge pieces on last layer
Permute edge pieces on last layer
Orient corner pieces on last layer
Permute corner pieces on last layer

1. WHITE CROSS

2. FIRST LAYER

L → F' U' F

B& R → R U R'

T → R U2 R'

~~B → R U R'~~

3 SECOND LAYER

B&R → U R U' R' F R' F' R

L – U' L' U L F' L F L'

~~B – U R U' R' F R F' R~~

4 YELLOW CROSS

LINE – CASE 1 – F R U R' U' F'

V – CASE 2 – F U R U' R' F'

DOT – CASE 3 – CASE 1 + U2 + CASE 2

CROSS – CASE 4 – DONE

5 YELLOW ~~EDGE~~ ~~CORNER~~ EDGE

CW R U2 R' U' R U' R'

CCW R U R' U R U2 R'

ADJ EDGE SWAP R U2 R' U' R U' R' + U2 CW + ~~[illegible]~~

\+ R U R' U R U2 R' + U2 CW OR CCW

6 YELLOW TOP

TOP COLOR ON R (R D R' D')(R D R' D')

TOP COLOR ON BACK (D R D' R')(D R D' R')

7 YELLOW CORNER

CW X R' U R' D2 R U' R' D2 R2

CCW X R2 D2 R U R' D2 R U' R

SWAP ADJ CORN (R' D2 R) U' (R' D2 R) U (R' D2 R) U (R' D2 R) U (R' D2 R) U' (R' D2 R) U'

SWAP DIA CORN (R' D2 R) U2 (R' D2 R) U2 (R' D2 R) U (R' D2 R) U2 (R' D2 R) U2 (R' D2 R) U'

A = R' D2 R

SWAP ADJ CORN A U' A U A U A U A U' A U'

SWAP DIA CORN A U2 A U2 A U A U2 A U2 A U'